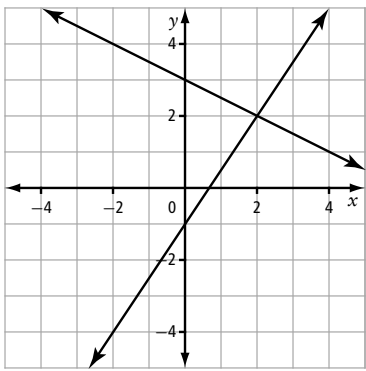
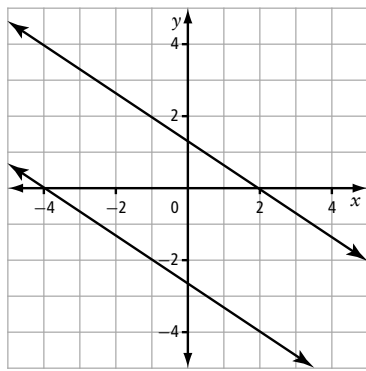
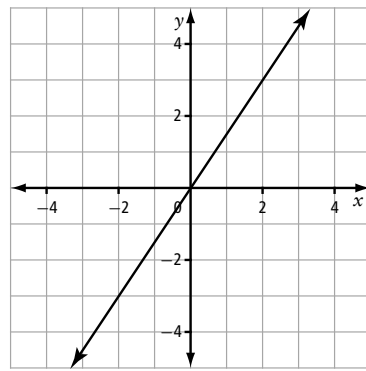


## 8.3 Number of Solutions for Systems of Linear Equations

### KEY IDEAS

- A system of linear equations can have one solution, no solution, or an infinite number of solutions.
- Before solving, you can predict the number of solutions for a linear system by comparing the slopes and  $y$ -intercepts of the equations.

Intersecting Lines	Parallel Lines	Coincident Lines
one solution	no solution	an infinite number of solutions
		
different slopes	same slope	same slope
$y$ -intercepts can be the same or different	different $y$ -intercepts	same $y$ -intercept

- For some linear systems, reducing the equations to lowest terms and comparing the coefficients of the  $x$ -terms,  $y$ -terms, and constants may help you predict the number of solutions.

Consider the system of linear equations

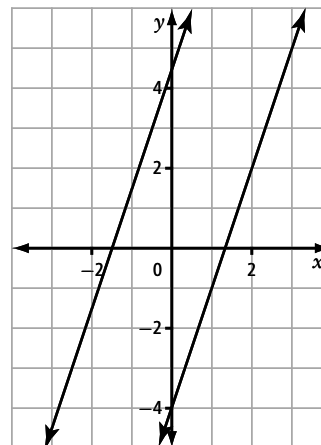
$$3x - y - 4 \text{ and } 6x - 2y = -9.$$

Rewrite the equations in slope-intercept form.

$$y = 3x - 4 \text{ and } y = 3x + \frac{9}{2}$$

The lines both have a slope of 3. Since they have different  $y$ -intercepts, the lines are parallel. Parallel lines have no intersection, so this system of linear equations has no solution.

Graphing the equations together shows that the lines are parallel and will not generate any solutions.



## Example

Four students are going to type the same lengthy essay. Shonna begins first. Anna and Kristian start together, a short time later, and type at an equal speed that is faster than Shonna's rate. James begins last and types at the same speed as Shonna. If the number of words typed by each student as a function of time could be represented by a linear equation, how many solutions would there be for each system of linear equations?

- a) Shonna and Anna
- b) Anna and Kristian
- c) Shonna and James

## Solution

- a) Although Anna started to type after Shonna did, Anna types more quickly. In time, Anna will catch up to and pass Shonna in terms of the number of words typed. There will be one point at which the numbers of words typed by Shonna and Anna are equal. Therefore, there is one solution.
- b) Since Anna and Kristian start at the same time and are typing at the same speed, they will have typed the same number of words at all times. Therefore, there is an infinite number of solutions.
- c) James is typing at the same speed as Shonna, but because he started typing after she did, he will never catch up. Therefore, there is no solution.

## A Practise

1. Predict the number of solutions for each system of linear equations. Justify your answers.

a)  $y = 4x - 1$   
 $y = 4x + 7$

b)  $y = 3x + 5$   
 $y = -3x + 5$

c)  $x + 2y = 7$   
 $5x + 10y = 35$

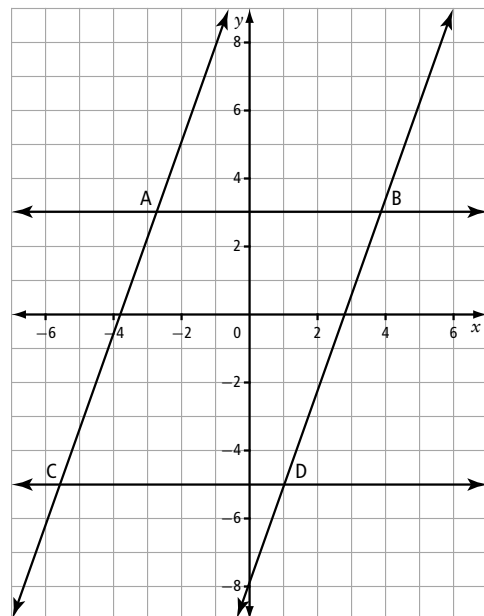
d)  $y = 2x + 3$   
 $y = 2x - 7$

e)  $2y = \frac{x}{2} + 3$   
 $3y = \frac{3x}{2} + \frac{9}{2}$

2. One equation of a linear system is  $2x - 3y = 8$ . Write a second equation so that the linear system will have
- a) no solution
  - b) one solution
  - c) an infinite number of solutions

3. The four lines on the graph intersect to produce parallelogram ABCD. Indicate the number of solutions for each system of linear equations.

- a) AB and CD
- b) AC and BD
- c) AB and AC
- d) BD and CD
- e) AD and BC



4. Graph each system of linear equations and indicate the number of solutions.
- a)  $6x + 2y = 10$   
 $y = -3x - 1$
- b)  $x + y = 9$   
 $x - y = 9$
- c)  $3y = x + 6$   
 $6y - 2x = 12$
5. Explain in words a method to correctly predict the number of solutions of a system of linear equations simply by looking at the equations.
6. A line is defined by the equation  $7x - 3y = 12$ . Determine the equation of a second line such that the system of linear equations has
- a) no solution  
b) an infinite number of solutions  
c) one solution
8. Service charges of two cell phone companies consist of a flat rate (a constant) and a rate per minute of use. If  $C$  represents the total cost and  $m$  represents the rate per minute, use values of your choice to create a system of linear equations that expresses the service charges of the companies where the system has
- a) one solution  
b) no solution  
c) an infinite number of solutions
9. Two students are using graphing technology to solve the system of linear equations  $y = \frac{3}{5}x + \frac{5}{2}$  and  $y = \frac{2}{3}x$ . The graphing calculator viewing window is set at X [0, 10, and 1] and Y [-2, 10, 1]. Antonio says that the system has no solution. Ling says it has one solution.
- a) Which student is correct? Why?  
b) How could the system of linear equations be solved without actually graphing the equations?

## B Apply

7. A real estate company is comparing the projected earnings,  $E$ , of several of its sales representatives. Projected earnings are based on current earnings plus a percent of sales,  $s$ , of each employee.

Employee	Current Earnings (\$)	Percent of Sales ( $s$ )
Jocelyn	1200	3.00
Mario	1000	4.50
Kendra	2000	3.00
Pavel	2000	3.00

Write a linear equation for the projected earnings of each sales representative. For which pair(s) of sales representatives could you create a system of linear equations that has

- a) no solution?  
b) an infinite number of solutions?  
c) one solution?

## C Extend

10. The Gold Coast Fishery pays fishers \$1.25/kg for Coho salmon and has a \$40 processing fee. The Salmon House pays fishers \$1.00/kg for Coho salmon and charges \$25 for processing.
- a) Determine a system of linear equations to model the earnings,  $E$ , in dollars, for fishers in terms of kilograms,  $k$ , of salmon delivered.  
b) Predict the number of solutions to the system.  
c) For what range of  $k$ -values should a fisher bring his or her salmon catch to Gold Coast Fishery? to The Salmon House?  
d) How would your answer to part c) change if The Salmon House raised its rate to \$1.25/kg to match its competitor?

11. Given the system of linear equations  $3x - 5y = 30$  and  $6x - 10y = C$ , for what value(s) of  $C$  is there

- a) no solution?
- b) an infinite number of solutions?

12. Given the system of linear equations  $2x + 6y = 12$  and  $Ax + 3y = 6$ , for what value(s) of  $A$  is there

- a) an infinite number of solutions?
- b) one solution?

13. Two 20-storey office towers are under construction. The first building is being erected at a rate of 2 storeys every 6 weeks. The height of each storey is 15 ft. Construction of the second tower began 4 weeks after the first. It is going up at a rate of 1 storey every 2 weeks. Each storey has a height of 12 ft.

- a) Determine a system of linear equations to model construction of the towers where  $h$  is the height of each building, in feet, and  $w$  is the number of weeks since construction first started.
- b) Predict the number of solutions to the system of linear equations.
- c) What is the solution to the linear system? What does the solution represent?

### D Create Connections

★14. Consider the statement “Determine two natural numbers having a sum of 20 and a difference of 10.”

- a) If a system of linear equations can model the statement, how many solutions will the system have?
- b) Compare the statement “Determine two natural numbers that have a sum of 10 and a difference of 20.” If a system of linear equations can model this statement, how many solutions will the system have?
- c) Graph each system of linear equations to confirm your answers for parts a) and b).

d) Why are the answers for parts a) and b) different?

15. There is an international dragon boat festival each year in Vancouver. Competitive racing takes place on False Creek over a course that is 500 m in length. Information about four dragon boats part way through one race is shown in the table of values.

Boat	Current Distance (m)	Current Speed (m/s)
A	220	3.1
B	206	3.4
C	198	3.6
D	230	3.2

Represent the data for each pair of boats using a system of linear equations, assuming that each boat continues at its current speed. What is the solution to each system? What does the solution represent?

- a) A and C
- b) A and D
- c) B and D

16. The volume of water,  $V$ , in two different pools as they are being filled is given by the equations  $V = 150 + 32.5t$  and  $V = 175 + 35t$ , where  $V$  is measured in litres and  $t$  is time, in minutes. Eva says that one pool has less water to start with and is filling more slowly, so the system of linear equations has no solution. Vince says that the lines have different slopes and must intersect, so there is one solution. Who is correct? Justify your answer.

17. A linear system of equations can be described in general as  $ax + by = c$  and  $dx + ey = f$ , where coefficients  $a, b, c, d, e, f \in \mathbb{I}$ . What is the relationship between the coefficients in the two equations such that the system has

- a) no solution?
- b) an infinite number of solutions?